Study Guide

1. Linear functions

Cumulative Practice Examination 1

[17 marks]

1 $\frac{3(2x-5)}{7} - 3 = \frac{4(5-2x)}{3} + \frac{1}{2}$

$9(2x-5) - 63 = 28(5-2x) + {}^{21}/_2$

$18x - 45 - 63 = 140 - 56x + {}^{21}/_2$

$74x - 108 = 140 + {}^{21}/_2$

$74x = 258\frac{1}{2}$

$148x = 517$

$x = \frac{517}{148}$

[3]

2 (a) $(-3, 1)$

parallel to $4y - 5x + 1 = 0$

$4y = 5x - 1$

$y = {}^5/_4 x - {}^1/_4$

gradient $= {}^5/_4$

$y - y_1 = m(x - x_1)$

$y - 1 = {}^5/_4(x+3)$

$4y - 4 = 5x + 15$

$4y - 5x - 19 = 0$

(b) midpoint of $(-2, 4)$ and $(6, 10)$

is: $\left(\frac{-2+6}{2}, \frac{4+10}{2}\right)$

$= (2, 7)$ $m = {}^{-1}/_3$

$\therefore$ $y - y_1 = m(x - x_1)$

$y - 7 = {}^{-1}/_3(x-2)$

$3y - 21 = -x + 2$

$3y + x - 23 = 0$ [4]

3(a) let x = amount in one container

y = amount in other container

$x + y = 800$ ..... (1)

$x = 3y - 200$ ..... (2)

(b) substitute (2) into (1)

$y + 3y - 200 = 800$

$4y = 1000$

$y = 250$

$\therefore$ $x = 3 \times 250 - 200$

$= 750 - 200$

$= 550$

$\therefore$ 250, 550 [4]

4 distance between 2 points

$\therefore$ $5 = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$\therefore$ $5 = \sqrt{(8-5)^2 + (-1-y)^2}$

$25 = 9 + (-1-y)^2$

$16 = (-1-y)^2$

$-1 - y = 4$ $\quad$ $-1 - y = -4$

$y = -5$ $\quad$ $y = 3$ [2]

5 (a) midpoint of AB

$\left(\frac{-3+7}{2}, \frac{11-7}{2}\right)$

$= (2, 2)$

(b) gradient $= \tan 135°$

$= -1$

(c) $y - 2 = -x + 2$

$y = -x + 4$

[4]

Cumulative Practice Examination 2

Multiple Choice [26 marks]

1 C	7 B
2 B	8 D
3 E	9 D
4 D	10 C
5 E	11 D
6 B	12 C

1 C

$3x - 2y + 8 = 0$

$2y = 3x + 8$

$y = \frac{3x}{2} + 4$

$\therefore m = 3/2 \quad c = 4$

2 B

$f: x \leq 1 \to R \quad f(x) = 5 - 2x$

$f(1) = 5 - 2$

$= 3$

$\therefore$ range $[3, \infty)$

3 E

$c = -2 \quad m = \frac{\text{rise}}{\text{run}}$

$= \frac{2}{3}$

$\therefore y = 2/3x - 2$

4 D

gradient $= \tan(90 - 55)$

$= \tan 35°$

5 E

$5x - 3y = 11$

$3y = 5x - 11$

$y = \frac{5x}{3} - 11/3$

$\therefore$ gradient $5/3$

6 B

$y + 3x - 5 = 0$

$y = -3x + 5$

$m = -3$

gradient of perpendicular is: $1/3$

passes through $(2,3)$

$\therefore y - 3 = 1/3(x - 2)$

$3y - 9 = x - 2$

$3y - x - 7 = 0$

7 B

$3(x - 2) = 4 - 2x$

$3x - 6 = 4 - 2x$

$5x = 10$

$x = 2$

8 D

$m = -2$ passes through $(-1, 3)$

$y - 3 = -2(x + 1)$

$y - 3 = -2x - 2$

$y = -2x + 1$

9 D

$2x - 3y = 1 \quad \ldots\ldots (1)$

$y = 3x + 2 \quad \ldots\ldots (2)$

substitute (2) into (1)

$2x - 3(3x + 2) = 1$

$2x - 9x - 6 = 1$

$-7x = 7$

$x = -1$

$y = -3 + 2$

$= -1$

10C

midpoint $(-2, 3)$ $(3, -5)$

is: $\left(\frac{-2+3}{2}, \frac{3+-5}{2}\right)$

11 D

distance between $(-1,3)$ $(4,7)$

$d = \sqrt{(4+1)^2 + (7-3)^2}$

12C

$(3,-5)$ $(6,5)$

$m = \frac{10}{3}$

$\therefore\ y+5 = \frac{10}{3}(x-3)$

$3y+15 = 10x-30$

$3y-10x+45=0$

Extended answer

1 98km/hr 80km next town

(a) $d_1 = 80 - \frac{98}{60}t$

$= 80 - \frac{49}{30}t$ [2]

(b) 105 km/hour in 10 minutes

travels 17.5 km

$\therefore$ distance to town is $80+17.5$

$= 97.5$ km

$d_2 = 97.5 - \frac{105}{60}t$

$= 97.5 - \frac{7}{4}t$ [2]

(c) 105 km/hour to travel 97.5 km

is 0.92857 of an hour

i.e. 56 minutes

$\therefore$ 12.56 p.m. [2]

(d) 98 km/hour to travel 80 km

is $\frac{80}{98} = 0.816327$

i.e. 49 minutes

$\therefore$ 12.49 p.m i.e. 7 minutes earlier

[2]

(e) $80 - \frac{49}{30}t = 97.5 - \frac{7}{4}t$

$\frac{7}{4}t - \frac{49}{30}t = 17.5$

$\frac{7}{60}t = 17.5$

$t = 150$ minutes

$\therefore\ d_1 = 80 - \frac{49}{30}\times 150$

$= -165$

i.e. 165 km past the town since d represents distance from town.

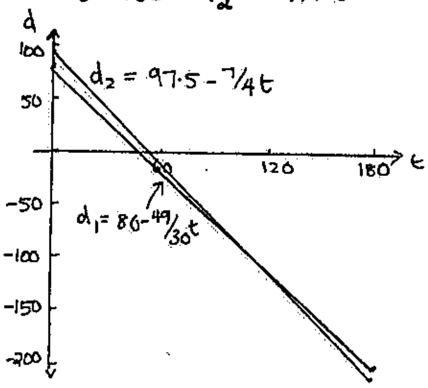
[2]

(f) $t=0$ $d_1 = 80$

$t=0$ $d_2 = 97.5$

$t=180$ $d_1 = -214$

$t=180$ $d_2 = -217.5$



distance apart is 3.5 km [3]

2. Quadratic functions

Cumulative Practice Examination 1

[17 marks]

1(a) $4x - 3y = 4 \quad \ldots\ldots (1)$

$2y - 5x = 2 \quad \ldots\ldots (2)$

$5\times(1)$ $\quad 20x - 15y = 20 \quad \ldots\ldots (3)$

$4\times(2)$ $\quad -20x + 8y = 8 \quad \ldots\ldots (4)$

$(3)+(4)$ $\quad -7y = 28$

$y = -4$

$\therefore 4x + 12 = 4$

$4x = -8$

$x = -2$

$\therefore x = -2 \quad y = -4$

(b) $5x - 3y = 13 \quad \ldots\ldots (1)$

$y = 2x - 3 \quad \ldots\ldots (2)$

sub(2) into (1) $\quad 5x - 3(2x-3) = 13$

$5x - 6x + 9 = 13$

$-x = 4$

$x = -4$

$\therefore y = -8 - 3$

$= -11$

$\therefore x = -4 \quad y = -11$

[5]

2 $y = x^2 + 3ax + 5$

intersects x-axis at only one point

$\therefore$ $x^2 + 3ax + 5 = 0$ has only one solution

i.e. $\Delta = 0 \quad b^2 - 4ac = 0$

$9a^2 - 20 = 0$

$9a^2 = 20$

$a = \dfrac{\pm\sqrt{20}}{3}$

[2]

3 $y = x^2$ dilated factor $\frac{1}{2}$ from x-axis

$\therefore y = \frac{x^2}{2}$

translated 4 units left

$y = \dfrac{(x+4)^2}{2}$

translated 2 units down

$y = \frac{1}{2}(x+4)^2 - 2$

$\therefore y = \frac{1}{2}(x^2 + 8x + 16) - 2$

$y = \frac{1}{2}x^2 + 4x + 8 - 2$

$y = \frac{1}{2}x^2 + 4x + 6$

$\therefore m = \frac{1}{2} \quad n = 4 \quad p = 6$

[3]

4(a) $y = 2x^2 + 3x + 4$

y-intercept : $x = 0$

$\therefore y = 4$

i.e. $(0, 4)$

(b) $x = \dfrac{-3 \pm \sqrt{9 - 4\times 2\times 4}}{4}$

no solution since can't find $\sqrt{-ve}$

$\therefore$ no x-intercepts

(c) $y = 2x^2 + 3x + 4$

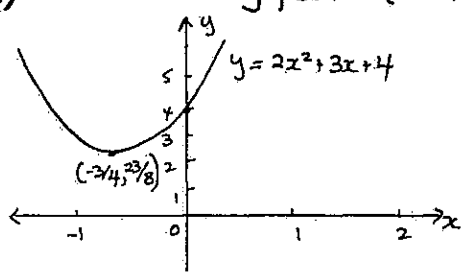
$= 2(x^2 + \frac{3x}{2} + 2)$

$= 2(x^2 + \frac{3x}{2} + (\frac{3}{4})^2 - (\frac{3}{4})^2 + 2)$

$= 2((x + \frac{3}{4})^2 - \frac{9}{16} + \frac{32}{16})$

$= 2(x + \frac{3}{4})^2 + \frac{23}{8}$

(d) $\therefore$ turning point $(-\frac{3}{4}, \frac{23}{8})$



[5]

5 $m^2 + 4m + 3 = 0$

(a) $(m+3)(m+1) = 0$

(b) $m = -1, \quad m = -3$

[2]

Cumulative Practice Examination 2

[26 marks]

Multiple choice

1 B	7 B
2 E	8 E
3 D	9 A
4 B	10 E
5 A	11 E
6 C	12 A

1B

$5x + 3y + 10 = 0$

$3y = -5x - 10$

$y = \frac{-5x}{3} - {}^{10}/_{3}$

$\therefore\ m = {}^{-5}/_{3} \quad c = {}^{-10}/_{3}$

2E

$f: [-3, 5) \to R \quad f(x) = 4x - 3$

$f[-3) = -15$

$f(5) = 17$

$\therefore$ range $[-15, 17)$

3D

$\frac{5x - 7y}{2} = 8$

$5x - 7y = 16$

$7y = 5x - 16$

$y = {}^{5x}/_{7} - {}^{16}/_{7}$

$\therefore$ gradient ${}^{5}/_{7}$

gradient of perpendicular ${}^{-7}/_{5}$

4B

$(-1, 2)$ and $(3, 4)$

$m = \frac{2}{4} = \frac{1}{2}$

$y - y_1 = m(x - x_1)$

$y - 2 = {}^{1}/_{2}(x + 1)$

$2y - 4 = x + 1$

$2y - x - 5 = 0$

5 A

$0 = 2(x + a)^2 - b$

$0 = 2x^2 + 4ax + 2a^2 - b$

$x = -1 \quad 0 = 2 - 4a + 2a^2 - b \quad — (1)$

$x = -5 \quad 0 = 50 - 20a + 2a^2 - b \quad — (2)$

$(2) - (1) \quad 0 = 48 - 16a$

$a = 3$

sub in (1) $\quad 0 = 2 - 12 + 18 - b$

$b = 8$

6 C

positive quadratic $(a, 0)\ (b, 0)$

$\therefore\ y = m(x - a)(x - b)$

$\therefore$ m could equal 2

$y = 2(x - a)(x - b)$

7B

$y = x^2$ translated left 3

$y = (x + 3)^2$ dilated by a factor of 2

$y = 2(x + 3)^2$

8E

$y = x^2$ dilated by ½

$y = {}^{1}/_{2}x^2$ translated left 3

$y = {}^{1}/_{2}(x + 3)^2$

9A

$y = -2x^2 + 3x - 5$

$= -2(x^2 - \frac{3x}{2} + {}^{5}/_{2})$

$= -2(x^2 - {}^{3x}/_{2} + ({}^{3}/_{4})^2 - {}^{9}/_{16} + {}^{5}/_{2})$

$= -2(x + {}^{3}/_{4})^2 - 3.875$

$\therefore$ range $(-\infty, -3.875]$

10E

negative quadratic

turning point $(a,0)$

y-intercept $(0,-b)$

$\therefore\ y = m(x-a)^2$

$m \times a^2 = -b$

$\therefore\ m = -b/a^2$

$\therefore\ y = -b/a^2 (x-a)^2$

11E

$(x+1)(x+6) = x^2 + 7x + 6$

$(x+2)^2 + a(x+2) + b$

$= x^2 + 4x + 4 + ax + 2a + b$

$= x^2 + (4+a)x + 4 + 2a + b$

$\therefore\ 4+a = 7 \qquad 4 + 2a + b = 6$

$a = 3 \qquad 4 + 6 + b = 6$

$b = -4$

12A

turning point $(2,3)$

negative quadratic graph

y-intercept $(0,-5)$

$y = a(x-2)^2 + 3$

$x = 0 \quad y = -5$

$-5 = 4a + 3$

$a = -2$

$\therefore\ y = -2(x-2)^2 + 3$

$y = -2(x^2 - 4x + 4) + 3$

$= -2x^2 + 8x - 8 + 3$

$y = -2x^2 + 8x - 5$

Extended answer

1 $p(t) = -0.5t^2 + 5t + 180$

(a) $t = 0 \quad p(0) = 180$

$= \$180$ /day [1]

(b) $p(2) = -0.5 \times 4 + 5 \times 2 + 180$

$= -2 + 10 + 180$

$= 188$ [1]

(c) $p(30) = -0.5 \times 30^2 + 5 \times 30 + 180$

$= -\$120$

i.e. lose \$120/day [1]

(d) $0 = -0.5t^2 + 5t + 180$

Using calculator $Y1 = -0.5t^2 + 5t + 180$

$t = 24.6214$ [2]

(e) $p(t) = -0.5(t^2 - 10t - 360)$

$= -0.5(t^2 - 10t + 25 - 25 - 360)$

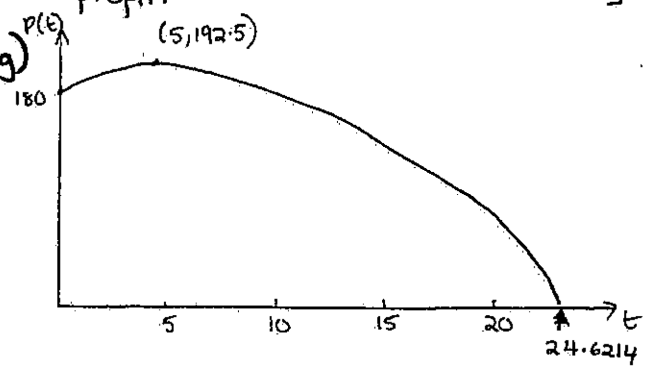
$= -0.5((t-5)^2 - 385)$

$= -0.5(t-5)^2 + 192.5$ [2]

(f) t value of maximum turning point is 5

$\therefore$ 5 additional staff will maximise profit. [1]

(g)



(h) Profit at least \$150/day

On calculator draw $Y2 = 150$ and find point of intersection $(14.219544, 150)$

$\therefore$ 14.22 or less additional staff will give at least \$150 profit/day.

[3]

3. Higher order polynomial functions

Cumulative Practice Examination 1

[17 marks]

1 $y = 3x^2 - 12x + 17$

$y = 3(x^2 - 4x + 17/3)$

$y = 3(x^2 - 4x + 4 - 4 + 17/3)$

$= 3((x-2)^2 + 5/3)$

$= 3(x-2)^2 + 5$

$\therefore$ turning point $(2, 5)$ [2]

2 (a) $y = 15 - 3(7-x)^3 \quad x \leq -2$

domain given $x \leq -2$

range: sub $x = -2$ $\quad y = 15 - 3(7+2)^3$

$= 15 - 3 \times 9^3$

$= 15 - 3 \times 729$

$= 15 - 2187$

$= -2172$

$\therefore$ range $y \leq -2172$ (positive cubic) [2]

(b) $y = -2(x-2)^4 + 3 \quad x > -2$

domain given $x > -2$

turning point is $(2, 3)$

$\therefore$ range is $(-\infty, 3]$ since it is a negative quartic $\cap$ [2]

3 $3x^3 - 20x^2 + 23x + 10 = 0$

Substitute $x = 1$ $\quad 3 - 20 + 23 + 10 \neq 0$

$x = -1$ $\quad -3 - 20 - 23 + 10 \neq 0$

$x = 2$ $\quad 24 - 80 + 46 + 10 = 0$

$\therefore x - 2$ is a factor

$$x-2 \overline{\smash{\big)}\, 3x^3 - 20x^2 + 23x + 10} = 3x^2 - 14x - 5$$

$3x^3 - 6x^2$

$-14x^2 + 23x$

$-14x^2 + 28x$

$-5x + 10$

$-5x + 10$

$3x^2 - 14x - 5 = \dfrac{(3x-15)(3x+1)}{3}$

$= (x-5)(3x+1)$

$\therefore 3x^3 - 20x^2 + 23x + 10 = (x-2)(x-5)(3x+1)$

$\therefore x = 2 \quad x = 5 \quad x = -1/3$

[2]

4(a) $2a^3 - 16b^3$

$= 2(a^3 - 8b^3)$

$= 2(a - 2b)(a^2 + 2ab + 4b^2)$

(b) $2(x-1)^3 - 16(x+2)^3$

$a = x - 1 \qquad b = x + 2$

$\therefore 2(x - 1 - 2(x+2))((x-1)^2 + 2(x-1)(x+2) + 4(x+2)^2)$

$= 2(x - 1 - 2x - 4)(x^2 - 2x + 1 + 2x^2 + 2x - 4 + 4x^2 + 16x + 16)$

$= 2(-x-5)(7x^2 + 16x + 13)$

5(a) $2x^4 - 12x^3 + 6x^2 + 52x - 48$

$= 2(x^4 - 6x^3 + 3x^2 + 26x - 24)$

sub into $x^4 - 6x^3 + 3x^2 + 26x - 24$

$x = 1 \qquad 1 - 6 + 3 + 26 - 24 = 0$

$\therefore x - 1$ is a factor

$$x-1 \overline{\smash{\big)}\, x^4 - 6x^3 + 3x^2 + 26x - 24} = x^3 - 5x^2 - 2x + 24$$

$x^4 - x^3$

$-5x^3 + 3x^2$

$-5x^3 + 5x^2$

$-2x^2 + 26x - 24$

$-2x^2 + 2x$

$24x - 24$

$24x - 24$

sub into $x^3-5x^2-2x+24$

$x=2 \quad 8-20-4+24 \neq 0$

$x=-2 \quad -8-20+4+24=0$

$\therefore x+2$ is a factor

$$
\begin{array}{r}
x^2-7x+12 \\
x+2\,\overline{\big)\,x^3-5x^2-2x+24} \\
\underline{x^3+2x^2} \qquad\qquad \\
-7x^2-2x \qquad \\
\underline{-7x^2-14x} \qquad \\
12x+24 \\
\underline{12x+24} \\
-
\end{array}
$$

$x^2-7x+12 = (x-4)(x-3)$

$\therefore 2x^4-12x^3+6x^2+52x-48$

$= (x-1)(x+2)(x-3)(x-4)$ [3]

(b) $81x^4-450x^2+625$

Let $A=9x^2$

$\therefore A^2-50A+625$

$=(A-25)(A-25)$

$=(A-25)^2$

$=(9x^2-25)^2$ [2]

Cumulative Practice Examination 2

[Marks: 37]

Multiple choice

1 C	8 A
2 C	9 C
3 D	10 D
4 E	11 B
5 A	12 D
6 B	13 D
7 E	

1 C

$c=2$

$m=\frac{\text{rise}}{\text{run}}$

$=\frac{-2}{3}$

$\therefore y=\frac{-2}{3}x+2$

2 C

$f: [-2,3) \to R$

$f(x)=-2x+3$

$f(-2)=4+3=7$

$f(3)=-2\times 3+3=-3$

$\therefore$ range $(-3,7]$

domain $[-2,3)$

3 D

$y-2=3(x+1)$

$y-2=3x+3$

$y=3x+5$

$\therefore m=3$ and y-intercept 5

4 E

$y=(x+a)(x+b)$

turning point $(-1,9)$

turning point is midway between x-intercepts which are $-a$ and $-b$.

only values possible are $a=2$ $b=-4$

5 A

$y=(x-a)^2$

turning point $(a,0)$

and positive quadratic

i.e. $\cup$

6 B

$y = (x-a)^3(x-b) \quad a, \text{and } b > 0$

there are two x-intercepts a and b ✓

y-intercept at ab × intercept is

$(-a)^3(-b) = a^3b$

point of inflection at $x = a$ ✓

as $x \to \infty$ $y \to \infty$ ✓ (it is a positive quartic)

since a and $b > 0$ a minimum turning point is in fourth quadrant ✓

7 E

$\left(\frac{5w}{2} - \frac{1}{w^3}\right)^3$

$= \left(\frac{5w}{2}\right)^3 - 3\left(\frac{5w}{2}\right)^2\left(\frac{1}{w^3}\right) + 3\left(\frac{5w}{2}\right)\left(\frac{1}{w^3}\right)^2 - \left(\frac{1}{w^3}\right)^3$

$= \frac{125w^3}{8} - \frac{75w^2}{4w^3} + \frac{15w}{2w^6} - \frac{1}{w^9}$

$= \frac{125w^3}{8} - \frac{75}{4w} + \frac{15}{2w^5} - \frac{1}{w^9}$

8 A

$6x^3+5x^2-12x+4$ has $ax-b$ and $2x^2+3x-2$.

Using CAS $(6x^3+5x^2-12x+4) \div (2x^2+3x-2) = 3x-2$

$\therefore a = 3 \quad b = 2$

9 C

$y = 2(x+3)^3 - 1 \quad x \geq 2$

$x = 2 \quad y = 2 \times 5^3 - 1$

$= 249$

$\therefore$ range $[249, \infty)$

10 D

$w = \frac{1}{4}mp^2$

$\frac{4w}{m} = p^2$

$\therefore p = \pm 2\sqrt{\frac{w}{m}}$

11 B

$x-3$ is a factor of x^3+ax^2-x-6

$\therefore 3^3 + 9a - 3 - 6 = 0$

$9a + 18 = 0$

$a = -2$

12 D

point of inflection $(-4, -3)$

y-intercept $(0, 1)$

$y = a(x+4)^3 - 3$

$x = 0 \quad 1 = a \times 4^3 - 3$

$a = 1/16$

$\therefore y = 1/16(x+4)^3 - 3$

13 D

turning point at $(a, 0)$

x-intercept $(b, 0)$

$\therefore y = (x-a)^2(x-b)$

$y = (a-x)^2(x-b)$

Extended answer

1(a) $y = -0.02(x^3 - 38x^2 + 441x - 1620)$

$x \in [0, B]$

Using CAS draw graph, B is the 2nd x-intercept

Finding 2nd zero gives $x = 20$

$\therefore B = 20$

(b) In the same way as for B

A is the 1st zero

$\therefore$ A = 9 [1]

(c) y-intercept is 32.4

$\therefore$ height is 32.4 [1]

(d) $x = 3$

$y = 12.24$

$\therefore$ height 12.24 [1]

(e) $y = 3$

$3 = -0.02(x^3 - 38x^2 + 441x - 1620)$

using CAS y2 = 3

finding points of intersection using appropriate bounds, gives

$x = 5.76$, 14 and 18.24 m to the right [2]

(f) $m = 4/5$ $c = 32.4$

$y = 4/5\,x + 32.4$ [2]

(g) $0 = 4/5\,x + 32.4$

$4/5\,x = -32.4$

$x = -40.5$

$\therefore$ 40.5 is the horizontal distance covered [1]

2 $a(x) = -0.5x^2 + 90$

(a) length of base is distance between x-intercepts.

$-0.5x^2 + 90 = 0$

$x^2 = 180$

$x = \pm 13.416$

$\therefore$ length of base $= 2 \times 13.416$
$= 27$ cm [2]

(b) $20 = -0.5x^2 + 90$

$140 = x^2$

$x = \pm\sqrt{140}$

i.e. (-11.83, 20) (11.83, 20)

$40 = -0.5x^2 + 90$

$100 = x^2$

i.e. (-10, 40) (10, 40)

$60 = -0.5x^2 + 90$

$60 = x^2$

$x = \pm\sqrt{60}$

i.e. (-7.75, 60) (7.75, 60)

$80 = -0.5x^2 + 90$

$20 = x^2$

i.e. (-4.47, 80) (4.47, 80) [4]

(c) lengths: $2\sqrt{140} = 23.7$ cm

$2\sqrt{100} = 20$ cm

$2\sqrt{60} = 15.5$ cm

$2\sqrt{20} = 9$ cm [2]

(d) 40 cm high $\therefore$ y-intercept 40

base length 30

$\therefore 0 = -a \times 15^2 + 40$

$a = 8/45$

$\therefore y = -8/45\,x^2 + 40$ [2]

(e) $15 = -8/45\,x^2 + 40$

$\dfrac{-25 \times 45}{-8} = x^2$

$x = \pm\sqrt{140.625}$

$= \pm 11.86$

$\therefore$ (-11.86, 15) (11.86, 15)

$30 = -8/45\,x^2 + 40$

$\dfrac{-10 \times 45}{-8} = x^2$

$x = \pm\sqrt{56.25}$

$= \pm 7.5$

$\therefore$ (-7.5, 30) (7.5, 30)

lengths 23.72 and 15

$\therefore$ length required is 38.72 cm [4]